Student Learning: What Has Instruction Got to Do With It?

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Abstract

A seemingly unending controversy in the field of instruction science concerns how much instructional guidance needs to be provided in a learning environment. At the one extreme lies the claim that it is important for students to explore and construct knowledge for themselves, which is often called discovery learning, and at the other extreme lies the claim that providing direct instruction is more beneficial than withholding it. In this article, evidence and arguments that support either of the approaches are reviewed. Also, we review how different instructional approaches interact with other instructional factors that have been known to be important, such as individual difference, self-explanation, and comparison. The efforts to combine different instructional approaches suggest alternative ways to conceive of learning and to test it.

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INTRODUCTION

> Learning results from what the student does and thinks, and only from what the student does and thinks. The teacher can advance learning only by influencing what the student does to learn.
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> Herbert A. Simon (1916–2001)

In the field of learning science, many efforts have been made to find optimal instructional conditions to promote student learning. As suggested by Herbert Simon, one of the founders of cognitive science, educators must understand how students learn and then translate this understanding into practice when designing a learning environment.

THE DEBATE OVER DISCOVERY LEARNING

A seemingly unending controversy in the field of instruction science concerns how much instructional guidance needs to be provided in a learning environment (Kirschner et al. 2006, Kuhn 2007, Tobias & Duffy 2009). Is it better to try to tell students what they need to know, or is it better to give students an opportunity to discover the knowledge for themselves? Conceiving of this problem as deciding whether to give or withhold assistance, Koedinger & Aleven (2007) called this issue the "assistance dilemma." The contrast between the two positions is best understood as a continuum, and both ends appear to have their own strengths and weaknesses. As a result, it is very difficult to find the right balance between the two extremes. The assistance dilemma is also related to the notion of "desirable difficulty" (Bjork 1994, Schmidt & Bjork 1992). Learning conditions that introduce certain difficulties during instruction appear to slow the rate of learning but often lead to better long-term retention and transfer than learning conditions with less difficulty (e.g., mixed- or spaced-practice effect).

In this article, we briefly review older evidence and then focus on the more recent studies on this issue. At one end of the continuum, it is argued that minimizing instruction encourages students to discover or construct knowledge for themselves by allowing students to freely explore learning materials (Bruner 1961, Papert 1980, Steffe & Gale 1995). This approach is based on a constructivist theory of learning, and Jean Piaget (1970, 1973, 1980) has been often referenced as a basis for constructivism. For instance, according to Piaget (1973), "To understand is to discover, or reconstruct by rediscovery, and such conditions must be complied with if in the future individuals are to be formed who are capable of production and creativity and not simply repetition" (p. 20). In a discovery learning environment, students are regarded as active learners and are given opportunities to digest materials for themselves rather than as passive learners who simply follow directions. Discovery-based approaches have been widely accepted as major teaching methods by teachers and educators with constructivist views of learning. In truth, however, most discovery learning environments actually involve some amount of guidance, so this approach might better be referred to as "minimal guidance."

Why has a discovery learning approach been widely advocated by many teachers and researchers? Reiser et al. (1994) summarized possible cognitive and motivational benefits of discovery learning. First, learning through discovery is believed to have several cognitive benefits such as the development of inquiry skills and the utility of learning from errors. Students are thought to have more meaningful understanding over rote learning due to great amounts of self-generating processes and from attempting to explain and understand their mistakes. Generating activity has been known to help long-term retention (Bobrow & Bower 1969, Lovett 1992, Slamecka & Graf 1978). However, these potential cognitive benefits can also be lost when trying to discover the knowledge. For instance, students may not be able to remember how they solved a problem after excessive floundering (Lewis & Anderson 1985). Also, this floundering tends to increase learning time, and students may never be able to discover the important principles that they are expected to learn (Ausubel 1964).

It has been also argued that discovery methods produce benefits for retention and transfer (Bruner 1961, Suchman 1961). To test this proposition, Gutherie (1967) trained students to decipher cryptograms with different forms of instructional methods. Gutherie compared students who were given explicit rules followed by problem practice with students who just tried to solve the problems and had to discover the rules. The discovery students did better on the transfer problems that required new rules. Similarly, McDaniel & Schlager (1990) found that the discovery method provided benefits when students had to generate a new strategy to solve a transfer problem but not when they could apply the learned strategy.

Second, discovery learning is believed to increase students' positive attitudes toward the learning domain (Bruner 1961, Suchman 1961). Learning through exploration allows students to have more control in a task, and this in turn fosters more intrinsic motivation. Intrinsically motivated students are known to find a learning task more rewarding and tend to do more productive cognitive processing in comparison with extrinsically motivated students (Lepper 1988). In addition, it is argued that discovery learning enables students to learn additional facts about the target domain. For instance, when preschool children were given an adult's pedagogical instruction without interruption, they tended to focus on only the target function of a toy that was shown by the adult (Bonawitz et al. 2011). In contrast, when the pedagogical demonstration was experimentally interrupted, children explored the function of the toy more broadly and were more likely to discover novel information.

Despite all the arguments for the benefits of discovery learning, the empirical evidence has been mixed at best. There now have been decades of efforts to dissuade educators of the benefits of discovery learning. For instance, in a 1968 summary of 25 years of research, Ausubel (1968, pp. 497–498) wrote:

> [A]ctual examination of the research literature allegedly supportive of learning by discovery reveals that valid evidence of this nature is virtually nonexistent. It appears that the various enthusiasts of the discovery method have been supporting each other research-wise by taking in each other's laundry, so to speak, that is, by citing each other's opinions and assertions as evidence and by generalizing wildly from equivocal and even negative findings.

Reviewing the more recent research in a paper provocatively titled "Should there be a three-strikes rule against pure discovery?", Mayer (2004, p. 17) concludes:

> Like some zombie that keeps returning from its grave, pure discovery continues to have its advocates. However, anyone who takes an evidence-based approach to educational practice must ask the same question: Where is the evidence that it works? In spite of calls for free discovery in every decade, the supporting evidence is hard to find.

Kirschner et al. (2006), in their similarly provocatively titled paper "Why minimal guidance during instruction does not work: an analysis of the failure of constructivist, discovery, problem-based, experiential, and inquiry-based teaching," come to a similar conclusion. However, as a sign that we do not have an all-or-none decision to make between pure discovery and direct instruction, Mayer (2004) concludes that it is important for students to construct their own knowledge and advocates "guided discovery" as the best way to achieve this. Also, Kirschner et al. acknowledge what they call the "expertise reversal effect" (Kalyuga 2007, Kalyuga et al. 2003), where experienced learners benefit more with low levels of guidance than with high levels of guidance. In this article, we review some of the previous studies that compare many different forms of direct instruction and discovery learning approaches to investigate the effect of different amounts of instructional guidance.

Instructional Design Based on a Constructivist Theory of Learning

Discovery learning is often aligned with what are called constructivist theories of learning, although as Anderson et al. (1998) note, there is a wide variety of constructivist positions, some of which are mutually contradictory. These various positions often suffer from a lack of clear operational definitions and replicable instructional procedures (Klahr & Li 2005). That qualification being noted, there have been several reports of successful constructivist designs in studies conducted in a school environment (e.g., Carpenter et al. 1998, Cobb et al. 1991, Hiebert & Wearne 1996, Kamii & Dominick 1998, Schwartz et al. 2011).

Cobb and his colleagues (1991) developed a set of instructional activities based on a constructivist view of learning and investigated the effect of the design to help children's mathematical learning. In project classrooms, children used a variety of physical manipulatives and worked in pairs to solve mathematical problems. After working in pairs, a teacher led a whole-class discussion, and the children talked about their interpretations and solutions. At the end of the project, students were given a standardized achievement test and another arithmetic test developed by the project group. The results showed that students in project and nonproject classrooms were not different in terms of the level of computational performance. However, the students in the project classrooms demonstrated higher levels of conceptual understanding than those in the nonproject classrooms. Although the ability to perform computational tasks seemed similar between the groups, closer analysis showed that nonproject students heavily depended on the use of standard algorithms. This dependency was consistent with the beliefs nonproject students demonstrated on the questionnaire asking about reasons for success in mathematics. The nonproject students believed it was important to conform to the solution procedures of others. The project students, however, believed collaborating (explaining their thinking to others) and understanding were important for success in mathematics.

In another study, Hiebert & Wearne (1996) traced children's development of understanding of mathematical concepts and computational skills over the first three years of school in two different instructional environments. To teach place value and multidigit addition and subtraction, they provided students with either conventional instruction or alternative instruction. In the alternative instruction classrooms, students were presented with contextualized problem situations and encouraged to represent problem quantities with physical materials (base-ten blocks) and written numbers. Using both representations, students had to develop solution strategies and were encouraged to discuss their strategies with the class. The standard algorithms for addition and subtraction were not formally taught; instead, students discussed how and why invented procedures did or did not work. On the other hand, in the conventional instruction classroom, the instruction was mainly guided by the textbook. Teachers taught students how to find the answer, and students worked individually. The use of physical manipulatives was not required, so they were not used much. Assessments of mathematical understanding supported superiority of the alternative instruction at the end of the third grade. Also, when the relations between conceptual understanding and computational skill were analyzed, different patterns of development were identified between the two groups. The majority of students who received alternative instruction seemed to show good understanding prior to or concurrent with good computational skill. On the other hand, conventionally instructed students tended to show correct computation skills before they developed good understanding.

As shown in these studies, discovering one's own procedures can lead to better understanding and transfer. Carpenter and his colleagues (1998) investigated how inventing strategies was related to the understanding of mathematical concepts and procedures. In this study, children were traced for three years to assess understanding of concepts and procedures on multidigit addition and subtraction. The study compared students who used an invented strategy with students who used a standard algorithm. Students who invented a strategy were able to use not only their own invented strategy (if asked to do so), but also the standard algorithm after they learned that. Invention students also showed better understanding of base-ten number concepts and better performance in a transfer task. On the other hand, the algorithm group showed significantly more buggy algorithms in their problem solving than did the invented-strategy group.

Kamii & Dominick (1998) also argued that teaching algorithms could harm understanding of multidigit computation. These investigators compared students who had been taught standard algorithms for multidigit computation with those who had not. The students who did not receive the algorithm instruction outperformed those who were taught algorithms. The algorithm-taught students also tended to produce more unreasonable answers, implying that they depended on the use of learned procedures and lacked a deep conceptual understanding about the computation procedures.

Practice Facilitates Successful Discovery Learning

The studies discussed above were conducted in the classroom, and in this setting it is difficult to control all the factors at play (nor would one want to). Some more-focused laboratory studies suggest that students learn better in a discovery learning environment. Interestingly, these studies are related to high levels of practice. When combined with high levels of practice or longer acquisition time, students appear to learn better in a discovery learning environment than in a direct instruction environment.

Brunstein et al. (2009) investigated how learning improves as students become more experienced through a series of learning sessions under different instructional conditions. Algebra-like problems were constructed in a novel graphical representation, and this novel format allowed studying of solving equations anew in college populations. Participants received different types of instructions according to experimental conditions; students were given verbal direction on general characterization of actions, direct demonstration on what to do, both, or none (discovery condition). Thus, in the discovery condition, participants were provided with none of the guidance, and they had to learn from the consequences of their actions. The results showed that although discovery students showed the worst performance in the early problems by making more errors, they performed best on the later problems by making fewer errors and taking shorter problem-solving time in comparison with the other-instructed students. Also, in the later phase of learning, students in the discovery learning condition showed the best performance, regardless of the position of the problems.

In their second study, Brunstein and her colleagues (2009) obtained quite different results when students only had one-quarter of the problems to practice on. About 50% of the participants in the discovery condition felt lost and wanted to quit the study, whereas none had quit in the first study. The remaining discovery participants did worse than those who received direct instruction. Comparing findings from the first and second study, the discovery learning approach appeared to be effective only with high levels of practice. Without this practice to consolidate their understanding, students in the discovery condition had an especially hard time in understanding problems.

Similar results were obtained by Dean & Kuhn (2006), who investigated the effects of direct instruction and discovery learning on teaching control-of-variable strategy (CVS) in the science domain. This study followed students' progress (acquisition and maintenance) over approximately six months. Fourth-grade students learned to design unconfounded experiments through computer-based inquiry tasks under one of the three conditions: direct instruction only (DI), practice only (PR), or a combination of instruction and practice (DI+PR). To design an unconfounded experiment, students were required to make a comparison by manipulating only one factor while setting all other conditions the same. In the DI condition, students received a single session of instruction without long engagement. In the PR condition, students freely practiced CVS with a computer program over 12 sessions without direct instruction. After general initial instruction, only direct instruction conditions (both DI and DI+PR) received a series of comparisons between two different experimental conditions and comments about whether the comparison was good or bad and an explanation as to why.

In a replication of the results of Klahr & Nigam (2004), direct instruction proved to be effective in an immediate assessment. However, in the tests given after the eleventh week, the advantage of direct instruction did not remain without further practice. In contrast, the practice group showed continually improving performance over time. Dean & Kuhn (2006, p. 394) conclude, "...direct instruction appears to be neither a necessary nor sufficient condition for robust acquisition or for maintenance over time." These results are consistent with the findings of Brunstein et al. (2009) in that although minimal guidance may not be effective in the earlier stage of learning, with high levels of practice, performance improves over time. However, Klahr and his colleagues (e.g., Klahr & Nigam 2004, Matlen & Klahr 2010) repeatedly found positive learning gains from direct instruction on teaching CVS; some of these studies are reviewed in detail in a later section.

The finding that discovery learning can be effective when accompanied with high levels of practice also suggests a new interpretation of a previous study (Charney et al. 1990) that investigated three different instructional approaches to teach college students to use a spreadsheet program with a command line interface. The three experimental conditions were tutorials, problem solving, and learner exploration. The tutorial condition was given the highest level of instruction and the exploration condition was given the lowest level of instruction, with the problem-solving condition in between. The results showed that the tutorial condition was worst and the problem-solving condition was best, with exploration coming in between. However, Tuovinen & Sweller (1999) criticized this study because time-on-task was not controlled. Alternatively, the longer training may have enabled minimal guidance to be effective, and the condition might have been superior to direct instruction even if direct instruction were given more time.

PROVISION OF DIRECT INSTRUCTION

Empirical Evidence on Superiority of Direct Instruction

Although the studies reviewed above might be seen as support for discovery learning, there is no lack of studies showing the superiority of direct instruction in many different domains, such as problem-solving rules (Craig 1956, Gagne & Brown 1961, Kittel 1957), programming (Fay & Mayer 1994, Lee & Thompson 1997), science (Chen & Klahr 1999, Klahr & Nigam 2004, Matlen & Klahr 2010, Strand-Cary & Klahr 2008), mathematics (Carroll 1994, Cooper & Sweller 1987, Sweller & Cooper 1985), and procedure learning (Rittle-Johnson et al. 2001). The success reported by tutoring programs in mathematics (such as the Cognitive Tutor) also supports the importance of providing instructional guidance in response to students' needs (Anderson et al. 1995, Koedinger et al. 1997).

Another good example showing the advantages of direct instruction is the series of studies Klahr and his colleagues have done on CVS. The original study (Chen & Klahr 1999) demonstrated that direct instruction was more effective than discovery learning in improving children's ability to design unconfounded experiments. However, this study has been criticized with respect to its epistemology because high CVS scores do not mean high level of authentic scientific inquiry (Chinn & Malhotra 2001). Following this criticism, Klahr & Nigam (2004) investigated effects of direct instruction and discovery learning on CVS in a more authentic context with third- and fourth-grade children. They found that, as in earlier studies, direct instruction was more effective than discovery learning. Moreover, they found that on the "far transfer" science fair assessment, the many children who mastered CVS in the direct condition performed just as well as the few children who mastered it in the discovery condition. Thus, contrary to one of the common claims for the superiority of discovery learning, their study demonstrated that far transfer did not depend on how children learned something, only that they learned it. Further investigations by Strand-Cary & Klahr (2008) have also bolstered the effectiveness of direct instruction compared with the discovery learning approach. These studies are particularly noteworthy because they show the superiority of direct instruction in a more complex domain and on transfer tasks, which differs from commonly held beliefs that direct instruction is only effective for rote skills and direct tests of knowledge.

Matlen & Klahr (2010) examined the effect of different sequences of high versus low levels of instructional guidance on teaching CVS to find an optimal temporal sequence of guidance. By crossing the amount of instruction with two separate training sessions, four different orderings of instructional guidance were tested. The four conditions were high+high, high+low, low+high, and low+low, depending on whether early and late practice provided high or low instructional guidance. High guidance provided direct instruction and inquiry questions, whereas low guidance provided only inquiry questions. The study found best learning and transfer when high levels of guidance were repeated in the early and late training sessions (i.e., high+high condition).

Example-Based Learning: Worked Examples

A particularly interesting class of studies compares example-based learning with problem-based learning conditions. In example-based learning (often referred to as the worked-example condition), learners are provided with a worked example to study. Worked examples are instructional tools to provide an expert's solution that students can emulate. They typically involve a problem statement, step-by-step solution steps, and a final answer to the problem (Atkinson et al. 2000, Renkl et al. 1998). Worked examples are usually alternated with problems. In contrast, in problem-based learning, learners simply practice solving problems after initial instruction. In this kind of manipulation, the worked examples are often characterized as providing a form of direct instruction (Kirschner et al. 2006), but one could reasonably argue that the examples only provide scaffolding for a discovery process. In any case, as reviewed below, example-based instruction is often effective.

Carroll (1994) examined the effect of worked examples as an instructional support in the algebra classroom. High school students learned to translate words describing mathematical situations into formal equations (e.g., writing "five less than a number" as "x–5") in either a worked-example condition or conventional practice condition. Initial instruction included three examples and three practice problems in both conditions. In the worked-example condition, students were given a worksheet with 12 pairs of problems, one example followed by one problem. In the conventional practice condition, students had to solve 24 problems in the same order but without any examples. Students from the worked-example condition outperformed those from the practice condition by showing fewer errors on both immediate and delayed posttest (both learned and transfer problems), decreased need for assistance from the teacher, and less time taken to complete the work. It had been reported earlier in several other studies that the traditional practice of problem solving was not as effective as example-problem pairs (e.g., Cooper & Sweller 1987, Paas & Van Merriënboer 1994b, Sweller & Cooper 1985, Trafton & Reiser 1993).

Tuovinen & Sweller (1999) also compared the exploration-learning condition with the worked-example condition in college students learning to use a database program. The superiority of worked examples was again reported, consistent with findings by Carroll (1994), but this time the advantage occurred only for inexperienced learners. Tuovinen & Sweller had participants rate the cognitive load they experienced (ratings of mental effort required to complete the task using a Likert scale). Cognitive load is considered a multidimensional construct that represents the load imposed on the cognitive system while performing a particular task and is often conceptualized with mental load, mental effort, and performance (Paas & Van Merriënboer 1994a,b). The exploration group reported experiencing higher cognitive load than the worked-example group, but again the difference was reliable only for students who had less experience. This suggests that providing examples is effective in part because it lowers cognitive load for challenged learners.

Zhu & Simon (1987) claimed that students could learn from worked examples and problem solving equally successfully and efficiently without lectures or other forms of direct instruction as long as examples and/or problems are appropriately arranged in a way that students do not make too much trial-and-error search. When learning from examples, students use the worked examples to induce the relevant procedures and principles and then apply these to new problems. On the other hand, when learning by doing (i.e., problem solving), students have to first generate appropriate worked examples for themselves. When a problem solver correctly solves a problem, the solution path becomes a worked-out example.

In Zhu & Simon's (1987) study, students learned to factor quadratic algebraic expressions and showed learning in the problem-solving condition comparable to the learning-from-examples condition without lectures. Three possible explanations were suggested for this successful learning in both conditions. First, students had already studied the meaning of factoring and factoring of integers, thus all students had the background knowledge that was prerequisite for learning current materials. Second, students were provided with procedures for checking the correctness of their answers. This reduced the probability of students making errors of induction from incorrect solutions. Third, all examples and problems were carefully arranged so that students had to attend to only certain aspects of problems. This could reduce inefficient trial-and-error search and in turn reduce working memory load.

Besides studies on worked examples, there is a great abundance of studies showing that providing an example is an effective instructional method. Some studies have compared providing an example with providing procedures or rules. For example, Fong et al. (1986) reported that students who were trained with examples performed as well as students who were trained with explicit rules in learning statistical concepts. Both training methods were equally effective at improving the quality of statistical reasoning. Training on both methods had an additional positive effect. Reed & Bolstad (1991) also found that it was more effective to provide both examples and written procedures than to provide either examples or procedures alone when teaching to construct equations for work situation word problems. However, in this study, providing examples was more effective than providing written procedures only.

INTERACTION WITH OTHER INSTRUCTIONAL FACTORS

Individual Difference (Expertise Reversal Effect)

As Tuovinen & Sweller (1999) demonstrated, the effectiveness of instructional method might differ based on a learner's previous experience or prior knowledge level. One instructional approach might be ideal for experienced learners but might not be effective, or might even be detrimental, for inexperienced learners or novices. The expertise reversal effect (Kalyuga 2007, Kalyuga et al. 2003) is an example showing the interaction between the level of the learner and level of instruction. This occurs when instructional guidance helps inexperienced learners, but it is not beneficial for experienced learners. The idea of aptitude-treatment interaction (Cronbach & Snow 1977) has a long history and has been tested by many researchers. For example, Campbell (1964) found that the high-aptitude group benefited from a self-direction learning method, whereas the low-aptitude group benefited from a programmed instruction method. Cronbach & Snow (1977) also reported that when learners were given an opportunity to process the information in their own way, only high-ability learners benefited; low-ability learners appeared to be handicapped by this. Aptitude-treatment interactions have been found in many domains including multimedia learning (Mayer & Sims 1994, Seufert et al. 2007), probability calculation (Renkl 1997), and logic programming (Kalyuga et al. 2001). This idea easily expands into the implementation of an adaptive instructional support found in intelligent tutoring systems, where instruction is adapted in response to the learner's progress (Anderson et al. 1995, Salden et al. 2010).

Shute (1992) argued that some studies perhaps failed to produce successful effects of instructional manipulations simply because their manipulations were having different effects on different ability groups. To test this idea, Shute (1992) investigated effects of two learning environments on mastering the basic principles of electricity and examined how effects differed depending on the learner's ability. In the rule-application environment, participants were given feedback that explicitly stated the variables and the relationships among those variables that were used to describe a principle for a given problem. In the rule-induction environment, the relevant variables were identified by feedback, but participants had to induce the relationships and generate their own interpretation. While there was no main effect of the learning environment on learning outcomes, there was a significant interaction between cognitive ability (an associative learning measure) and the learning environment. Also, the interaction pattern was different for different learning outcome measures. For declarative knowledge acquisition, rule-induction was more effective for high-ability learners, but rule-application was more effective for low-ability learners. In contrast, for procedural skill acquisition, high-ability learners benefited more from the rule-application environment, and low-ability learners showed poor learning outcome regardless of the type of learning environment.

A possible explanation for this intriguing interaction is that learning is enhanced when there is a good match among learning environment, outcome measure, and cognitive ability. For example, one will acquire more robust declarative representations if one induces the rules than if one just applies rules. Because the high-ability learners possessed relevant cognitive skills, they were able to understand concepts and formulate a rule in the rule-induction condition. However, the low-ability learners lacked these cognitive skills to induce the rules and were able to acquire more declarative knowledge when the rules were explicitly provided in the rule-application condition. In contrast, the rule-application environment supports acquisition of procedural skills. In the rule-application condition, the high-ability learners promptly applied rules and procedures without a demanding induction process. This allows more opportunity to practice procedural skills than in the rule-induction condition. However, the low-ability learners never proceduralized necessary skills within the training time because of their deficient skills, and they performed poorly regardless of the type of learning environment.

Kalyuga et al. (2001) examined the interaction between instructional guidance and learners' knowledge level. In their study, trade apprentices from manufacturing companies were given general instruction on programmable logic controller programs for relay circuits and then given experimental training sessions. Participants had either pure problem-solving practice or a mixed worked-example and problem-solving practice. They found that students benefited less from worked examples as they mastered more material. The worked-example group showed performance superior to that of the problem-solving group in the early phase of the learning, but the difference was reversed in the end of the learning.

The findings indicate that levels of learner knowledge interact with levels of instructional guidance and suggest that students may learn better if different instructional methods are used, depending on the learner's experience through the acquisition/learning phase. According to this rationale, Renkl et al. (2000) proposed the combination of two instructional methods (worked example and problem solving) by presenting examples in the early stage of learning and then presenting problems in the later stage of learning. Renkl and his colleagues (2002) tested this proposal and tested a fading procedure against traditional example-problem pairs. In the fading procedure, a complete example is presented first, and then increasingly more incomplete examples are presented by omitting solution steps. Finally, a complete problem is presented. The study found positive effects of the fading procedure on near-transfer items. Atkinson et al. (2003) replicated this fading-out example effect by comparing example-problem pair learning with the backward fading procedure (where the last solution steps are omitted first). Schwonke et al. (2007) also found that tutored problem solving combined with gradually faded examples led to a better transfer performance than did tutored problem solving alone. However, all of these studies employed a fixed fading scheme, and the fading schedule was not adapted to the student's learning. Schwonke et al. (2007) suggested the fading example would be more beneficial if worked-out steps were to fade adaptively for each individual learner.

Following this suggestion, Salden et al. (2010) examined the effects of the fading of worked-out examples that occurred either fixedly or adaptively within the Geometry Cognitive Tutor. In the fixed fading condition, students were initially provided with complete worked examples, the problems with example steps gradually faded, and at the end they received pure problems according to the fixed schedule. In the adaptive fading condition, an individual student's mastery of geometry theorems estimated by a Bayesian knowledge-tracing algorithm (Corbett & Anderson 1995) was used to decide when a worked-out step should be faded. The results showed that the adaptive fading of worked-examples led to a better performance on delayed posttests than did the fixed fading of worked-examples or the standard tutored problem-solving practice.

Self-Explanation

Researchers have also been interested in how the discovery versus instruction dichotomy interacts with other instructional factors. For instance, the effect of self-explanation has been investigated along with the amount of instructional guidance in several studies (e.g., Atkinson et al. 2003, Rittle-Johnson 2006). The self-explanation effect occurs when students try to explain the example solutions to themselves and then learn more than those who do not. The original study on self-explanation was performed by Chi et al. (1989) and has been followed up in many laboratory experiments (e.g., Renkl et al. 1998, Siegler 2002) and classroom studies (e.g., Aleven & Koedinger 2002, Hausmann & VanLehn 2007). Self-explanation activity is thought to help learning by causing the generation effect when students generate their own explanations. For example, Hausmann & VanLehn (2007) showed that students who were prompted to generate their own explanations for examples showed greater learning gains than those who were prompted to paraphrase provided explanations for the same examples. A follow-up study by Hausmann et al. (2009) examined effects of different types of self-explanation prompts and found that justification and step-focused prompts benefited more from studying examples than did the meta-cognitive prompts. It appears that the first two prompts facilitate the acquisition of problem schema when students generate justification for each solution step.

Chi and her colleagues (1989) divided students into "good" and "poor" categories based on their problem-solving scores and analyzed the quality of their self-generated explanations collected using the think-aloud method. The analysis revealed that good and poor students differed not only in the amount of verbal protocols they provided, but also with respect to the quality of their explanations. Good students produced more explanations, more idea statements, and more statements that identified their own misunderstandings. While solving problems, good students tended to make more specific inquiries to examples they studied earlier when they had difficulty. However, in this study, studying time was not controlled. Good students actually spent more time to study the worked-out examples than did the poor students. Therefore, it was not clear whether more time-on-task or better self-explanation achieved the successful learning.

This kind of different characterization of self-explanation from good versus poor students was also reported by Renkl (1997), who controlled time-on-task. With verbal protocol analysis, four groups of participants were identified with respect to self-explanation styles, independently from achievement data. The four styles were passive, superficial, principle based, and anticipative. Passive explainers generated a poor quality of self-explanations and did not inspect many examples. Superficial explainers inspected many examples, but they spent relatively little time when studying each example. These two groups showed worse performance on the posttest in comparison with principle-based and anticipative reasoners. Principle-based explainers attempted to emphasize the meaning and goal of operators and elaborate on the underlying principles of examples. Anticipative reasoners appeared to use an example to test their problem solving. This group of people anticipated the next step of the example solution and moved on to the next page to check whether their anticipated solution step was actually correct or not. Pretest score differences suggest that different levels of prior knowledge affected the preference of explanation style. Anticipative reasoners had a relatively high level of prior knowledge, whereas principle-based explainers had a low level of prior knowledge.

Aleven & Koedinger (2002) showed that prompting for self-explanation was beneficial for learning in a class environment by implementing it as part of the Cognitive Tutor Geometry course. Students practiced problem solving in an intelligent tutor program either with or without a prompt to explain solution steps. In the self-explanation condition, students had to type the name of the problem-solving principle that justified the solution step, and the tutor then provided feedback on the correctness of the typed principle. Students who were prompted to explain their solution steps showed greater understanding and better transfer performance than those who were not asked to explain steps. Students trained without self-explanation prompts appeared to show shallow procedural knowledge.

Rittle-Johnson (2006) investigated whether promoting self-explanation is effective in combination with either direct instruction or discovery learning conditions. Third- through fifth-grade children learned to solve mathematical equivalence problems. Children often understand the equal sign (=) as an operator signal that gets the answer rather than understanding it as a relational symbol meaning that two sides of the equations are the same (Baroody & Ginsburgh 1983, Rittle-Johnson & Alibali 1999). The four conditions were constructed by crossing two factors, instruction type (instruction versus invention) and self-explanation prompt (self-explanation versus no explanation). For the instruction groups, a teacher taught a correct add-subtract procedure for solving problems. For the invention groups, no instruction was provided, and instead children were simply told to think of a new way to solve the problem. For self-explanation groups, children were given an additional screen showing two different answers from two children at another school: one correct and one incorrect answer. The children were asked to explain how the answers were obtained by the other children and why each answer was correct or incorrect. For the no-explanation group, the additional screen was not provided. The results showed that self-explanation and instructional type did not interact; rather, they simply had an additive effect on learning in that both self-explanation and direct instruction helped children learn a correct procedure.

Cognitive Load Theory and Designing an Effective Worked Example

As we have reviewed, it is often found that conventional problem-solving practice is not an ideal instructional method (e.g., Cooper & Sweller 1987, Sweller & Cooper 1985), and worked examples have been suggested as a better instructional approach (e.g., Carroll 1994, Paas 1992, Renkl 2002, Tuovinen & Sweller 1999; for review, see Atkinson et al. 2000). Also, worked examples are especially effective for inexperienced learners. Because levels of knowledge tend to interact with levels of instructional guidance, some variants of this instructional method, such as the fading procedure, were suggested to maximize its effect. What makes the worked-example approach effective for inexperienced learners but not for experienced learners? One of the most discussed explanations is the cognitive load theory (Sweller 1988). Humans have limited working memory capacity (Baddeley 1992, Miller 1956), and problem solving requires using this limited working memory. Therefore, solving a problem involves high working memory demands (e.g., to keep track of where one is in a search space), and most of the working memory resources are consumed for this activity rather than for supporting learning.

Sweller (1988) elaborates this idea in terms of learning domain schemas. Problem solving and schema acquisition are both demanding of the mechanisms of selective attention and limited working memory capacity. Problem solvers tend to focus on reducing the difference between the current state and the goal problem state and try to find the right operators to reduce this difference. This focus on specific differences does not help construct the general schemas for a domain. Moreover, learners often flounder as they search for the right operators and lose touch with the important information. However, when direct instruction or a worked example is given, learners do not need to use their working memory resources for an inefficient search and instead can use them to learn the essential relations between problem-solving moves. If a means-ends strategy prevents learners from acquiring schemas, reducing or eliminating goal specificity helps enhance schema acquisition by eliminating the possibility of using a means-ends strategy to solve a problem.

In some studies, when a conventional specific goal was replaced by a nonspecific goal, learning was actually enhanced (Miller et al. 1999, Sweller & Levine 1982, Sweller et al. 1983).

Sweller and his colleagues (1998) further discuss three types of cognitive load: intrinsic, extraneous, and germane. Intrinsic cognitive load cannot be altered by instructional design because it is intrinsic to the learning material, whereas extraneous and germane cognitive load can be reduced or induced by instructional design. Intrinsic load is the inherent level of difficulty that is directly associated with the material. When learning material involves more elements to consider (e.g., learning to multiply out the denominator in an equation), it has a more intrinsic load than when it does not (e.g., memorizing Fe is the symbol for iron). Extraneous load is often a result of poor instructional design and consumes one's working memory capacity with irrelevant activity, whereas germane load is a result of mental efforts that contribute to schema construction. Thus, an appropriate instructional design should reduce the extraneous cognitive load while inducing the germane cognitive load within working memory capacity. For example, Paas & Van Merriënboer (1994b) demonstrated that providing worked examples (in comparison to problem solving) enhanced learning by reducing the extraneous load, and that introducing variability in examples had positive effects only when the extraneous cognitive load was reduced.

According to cognitive load theory, the expertise reversal effect (Kalyuga et al. 2003) is an example of this phenomenon. Novices do not have sufficient prior knowledge to organize key information provided in the problem. Therefore, they have to do unproductive problem-solving searches. However, as learners become more experienced, the knowledge is stored in long-term memory, and the well-organized knowledge structures help overcome working memory limitations. This difference in working memory capacity between experienced and inexperienced learners results in different beneficial effects from worked examples.

Can direct instruction or worked examples harm learning by increasing working memory load? Several studies have shown that this is actually possible and suggest that instruction needs to be designed to reduce extraneous working memory load so that learners can focus on essential learning activities. Learning materials often are presented in various modalities such as text and diagram. When multiple sources of information are presented together, learners need to integrate corresponding representations. Difficulty in integrating separate sources of information causes split attention (Tarmizi & Sweller 1988) and prevents learners from constructing a relevant schema by increasing working memory load. Chandler & Sweller (1991) demonstrated that in the design of instruction, a diagram alone was more effective than a diagram with text. Also, presenting text in both visual and auditory format was less effective than in auditory format only (Craig et al. 2002, Kalyuga et al. 2000, Mayer et al. 2001). However, a dual-mode presentation is not always worse than a single-mode representation. If integration of different formats of information does not create a working memory burden, it can be effectively used. One of the major reasons that a word-plus-diagram presentation is not superior to a stand-alone diagram is the extensive visual search it requires. In essence, people need to find which part of the text corresponds to which part of the diagram. Based on this idea, Jeung et al. (1997) tried to reduce the visual search by using visual flashes to identify the part of a diagram to which the auditory text was referring. This technique proved to enhance learning. The importance of visual cueing also has been reported in the domain of animations by several researchers (Boucheix & Lowe 2010, de Koning et al. 2010).

Koedinger and his colleagues (2010) provide an alternative account for the worked-example effect and the expertise reversal effect. They argue that problem-solving practice is not effective for novice learners, not because of exhausted working memory capacity (as argued by cognitive load theory), but rather because of lack of environmental support for filling in their knowledge gaps. Worked examples provide more input than problem solving and therefore offer beginning learners a better opportunity for the induction and sense-making process. In contrast, advanced learners need refinement and fluency building, and these skills are better provided by problem-solving practice than worked examples.

Effects of Comparison in Learning by Worked Examples

Many researchers have emphasized the importance of comparison for learning and transfer (e.g., Gentner et al. 2003, Gick & Holyoak 1983). The National Council of Teachers of Mathematics standards also emphasize the importance of comparing solution methods as an instructional practice (Natl. Counc. Teach. Math. 1989, 2000). Students are encouraged to share and compare their solution methods with their classmates. This comparing method has been used as one of the instructional changes in many constructivism-based classrooms (e.g., Cobb et al. 1991, Hiebert & Wearne 1996).

Rittle-Johnson and her colleagues investigated when and how comparison helped learning in mathematics with school-age children in a series of studies. Rittle-Johnson & Star (2007) had seventh-grade children learn to solve multistep linear equations [e.g., $3(x + 1) = 15$] under one of the two different conditions, either the comparison or sequential condition. In the comparison condition, students were provided with sets of two worked examples illustrating different solutions for the same problem and were encouraged to compare and contrast the two examples. The solution steps of the two worked examples were mutually aligned together on the same page, and each step of the solutions was labeled (e.g., distribute, combine) as well. In the sequential condition, students studied the identical worked examples, but each worked example was presented on a separate page. Also, students were prompted to reflect on the solution of each example. After two days of intervention, students were tested on conceptual knowledge, procedural knowledge, and procedural flexibility. The results showed that students from the comparison condition gained more procedural knowledge and flexibility than those from the sequential condition, but there was no difference in conceptual knowledge between the two groups. Students who compared alternative solution methods were more likely to use the more efficient nonconventional methods and were better able to transfer their methods to novel problems.

Although the comparison proved to facilitate learning for multiple solution methods in mathematics, it is also important to know when and how comparison facilitates learning. Rittle-Johnson & Star (2009) showed that the effectiveness of comparison actually depended on what types of things were compared. Eighth-grade children learned to solve equations using worked examples in one of three different comparison conditions: comparing solution methods, comparing problem types, and comparing equivalent problems. The first condition was identical to the comparison condition used in the previous study by Rittle-Johnson & Star (2007) and involved learning multiple solution methods for one problem (i.e., one problem with two solution methods). In the comparing problem types condition, students learned to solve different problems with the same solution method (i.e., two different problems with one solution method). In the comparing equivalent problems condition, students learned to solve equivalent problems with the same solution method (i.e., two equivalent problems with one solution method). The posttest results showed that comparing solution methods was more effective for both conceptual knowledge and procedural flexibility than comparing problem types or comparing equivalent problems. Therefore, the benefits of comparison appear to depend on how worked examples differ.

Rittle-Johnson and her colleagues (2009) further examined the importance of prior knowledge in learning from comparison. Students were divided into two groups based on whether or not they attempted algebraic methods in a pretest. The results showed that students who attempted algebraic methods at pretest (high prior knowledge group) benefited most from comparing solution methods, but students who did not attempt algebraic methods at pretest (low prior knowledge group) were harmed by comparing solution methods. Students appeared to need sufficient prior knowledge in order to benefit from comparing alternative solution methods. When students do not have enough prior knowledge, two simultaneously presented worked examples are simply two unfamiliar examples, and the comparison activity just adds to the working memory load. In contrast, when students have enough prior knowledge, they can make an analogy from a familiar example to an unfamiliar example, and the comparison activity can be appropriately handled by their working memory resources.

Effects of Instructional Explanations in Learning by Worked Examples

In the studies reviewed, there was considerable variation in how much explicit (verbal) instruction accompanied the examples. Will students learn better from worked examples with instructional explanations or will they learn better if they are given only worked examples without instructional explanations? A large number of studies compared learning by worked examples with instructional explanation and without instructional explanation. In some studies, effects of receiving versus generating explanations have been compared when students learn with worked examples. There are both positive (e.g., Atkinson 2002, Lovett 1992, Renkl 2002) and negative (e.g., Ward & Sweller 1990) effects of the provision of instruction. Some studies showed neutral effects as well (e.g., Gerjets et al. 2006).

Lovett (1992) investigated the benefits of generating and receiving information when learning by problem solving versus when learning by example. Students learned to solve probability calculation problems in one of four experimental conditions. By crossing instruction type (worked example versus problem solving) with explanation type (instructional explanation versus self-explanation), four conditions were constructed. All groups of students demonstrated comparably good performance on the near-transfer test. A far-transfer test, however, showed a significant interaction between the instruction type and explanation type. When students learned by worked examples, students who received instructional explanation outperformed those who did self-explanation. On the other hand, when students learned by problem-solving practice, the pattern of results was reversed. Students who did not receive instructional explanation (i.e., self-explanation) showed better performance than those who did receive instructional explanation. These results were explained by the consistency of source information. In the example-based learning, the source of the solution is the experimenter, whereas in problem-based learning, the source of the solution is the subject. Likewise, the source of elaboration is experimenter for instructional explanation and the subject for self-explanation. When there are inconsistent information sources, subjects have to integrate their own information with the experimenter's, and this might have increased cognitive load and weakened their problem memories. Whether this is the correct explanation or not, the benefits of providing instructional explanations were found only when students learned in worked-example conditions.

Renkl (2002) also demonstrated that instructional explanations had a positive effect on learning by worked-out examples. In this study, he first compared favorable features of self-explanation and instructional explanation and then created a learning environment by combining and maximizing their respective advantages. Relative to self-explaining activities, instructional explanations are not usually adapted to the prior knowledge of individual learners and are more likely to be provided without consideration of students' ongoing cognitive activity[1] (Renkl 2002). Also, students lose an

[1]Intelligent tutoring systems such as Cognitive Tutor often provide instructional explanations that are adapted to an individual learner's knowledge level (Corbett & Anderson 1995).

opportunity to benefit from the generation effect (Hausmann & VanLehn 2007, Lovett 1992). However, instructional explanations have the important benefit of correctness. Students are known to generate incorrect self-explanations and then suffer from the illusion of understanding (Chi et al. 1989). Also, instructional explanations help students overcome comprehension problems that they cannot solve for themselves. Renkl developed a learning environment based on this analysis. By including or excluding an instructional explanation button, student teachers learned to solve probability calculation problems under one of two different conditions. Instructional explanations had a positive effect in far transfer but not near transfer. He also found that the explanations were used mostly by participants with low levels of prior knowledge.

Catrambone (1998) found that the provision of a simple label for worked examples helps learning. When a label is provided for a group of solution steps that go together, students attempt to self-explain the purpose of the grouped solution steps and organize them with subgoals. This is consistent with the finding that students can understand the general rationale of problems better when problem solutions are broken down into smaller meaningful solution units (i.e., modular examples) rather than when examples focus on problem categories and their associated overall procedures (Catrambone 1994, Gerjets et al. 2004).

Although several studies support the critical role of instructional explanations in example-based learning, this effectiveness does not seem to be guaranteed. For instance, Gerjets et al. (2006) reported no effect of providing instruction on learning probability calculation problems. Although the amount of instruction had no effect on test performance, students who received high levels of instruction erroneously felt more successful at learning than those who received low levels of instruction. As a matter of fact, more instructional explanations increased studying time; thus, less-elaborated example-based instruction was more efficient than more-elaborated example-based instruction.

Provision of instructional explanation sometimes even produces negative effects when added in an inappropriate way. Through a series of experiments, Ward & Sweller (1990) demonstrated that when the instructional explanation failed to direct attention appropriately, it failed to reduce cognitive load and thus was not effective. In one experiment, tenth-grade students learned geometric optics problems in one of three different experimental learning conditions: conventional worked example, split-attention worked example, and conventional problem. In the split-attention worked-example condition, extra textual explanation was added, but not in an integrated format. This group was no better than the conventional problem-solving group, and both were worse than the conventional example group.

Negative effects of providing instructional explanations also seem to occur by reducing self-explanation activities. Schworm & Renkl (2006) investigated how generating explanations interacts with receiving explanation in a domain of instructional design. Student teachers learned to design effective worked-out examples for high school students in several domains including geometry and physics. By crossing presence of self-explanation (self-explanation versus no self-explanation) with presence of instruction (instructional explanation versus no instruction), four different learning conditions were constructed. Participants were given initial instructions on basic principles of worked-out example design, and they studied solved example problems. Schworm & Renkl (2006) found that instructional explanations hurt learning when participants generated self-explanations but helped learning when they did not. There was reduced self-explanation activity in the presence of instructions.

Given contradictory evidence, it is hard to draw a conclusion on the role of instructional explanations added on worked examples. To address this issue, Wittwer & Renkl (2010) conducted a meta analysis (see also Wittwer & Renkl 2008). In order to investigate whether instructional explanations support example-based learning, 21 experimental studies were reviewed and analyzed with various moderating factors, and four major conclusions were reached. First, the overall effect of instructional explanation for example-based learning appears to be minimal. Although the provision of instructional explanation led to significantly better learning outcome than no instructional explanation, the effect size was small ($d = 0.16$). The benefit of instructional explanations was greater when the control condition was not supported by self-explanation. Second, instructional explanations were more effective for acquiring conceptual knowledge rather than procedural knowledge ($d = 0.36$). Third, the effectiveness of instructional explanations differed based on the learning domain. In mathematics, it had significantly positive effects ($d = 0.22$), but the effects were not clear in other domains including science and instructional design. Fourth, instructional explanations were not necessarily more helpful than the other supporting methods such as self-explanation. This analysis showed that prompting for self-explanation was as effective as adding instructional explanations for example-based learning.

COMBINING DISCOVERY LEARNING AND DIRECT INSTRUCTION APPROACHES

Invention Activity Followed by Direct Instruction

Both discovery learning and direct instruction approaches are known to have unique advantages (Koedinger & Aleven 2007), and there have been several attempts to combine discovery learning approaches with direct instruction approaches. For example, worked examples and problem-solving practice were successfully combined using a fading procedure (e.g., Atkinson et al. 2003, Renkl et al. 2002, Salden et al. 2010). When transitioning from worked examples to problem-solving practice depending on the stage of learning, it led to successful learning. There have been other attempts to combine invention activity with a follow-up direct instruction (e.g., feedback, lecture, and text), and several studies have shown that this combined method is more beneficial than administering only direct instruction and practice without invention activities (e.g., Kapur 2011, Kapur & Bielaczye 2011, Schwartz & Martin 2004, Schwartz et al. 2011).

For example, Schwartz & Martin (2004) demonstrated that a student's invention activities appeared inefficient because they failed to generate canonical solutions, but when a subsequent instruction was embedded in a test, these students actually did better than those who were directly taught and had to practice without invention activities. In this study, ninth-grade algebra students studied statistical concepts under one of two instructional conditions (invention versus tell-and-practice) and then were tested under one of two test conditions (presence versus absence of a worked example embedded into the test). Students in the tell-and-practice condition performed on par, regardless of whether there was a worked example embedded in the test. In contrast, students in the invention group outperformed these two groups, but only when there was a worked example embedded in the test. This study shows that while one instructional approach may look ineffective, the efficacy of the approach may actually be hidden in what Bransford & Schwartz (1999) call a sequestered problem-solving paradigm. This is when participants are sequestered for tests of their learning to prevent them from possible exposure to other sources that may positively or negatively affect their performance in experiments. In contrast, in a preparation for future learning paradigm (Schwartz & Bransford 1998), learners are tested not based on whether they can generate a finished product, but rather based on whether they are prepared to learn to generate a new product.

Schwartz and his colleagues (2011) also reported similar findings with adolescent students. Students learned a concept of density under either a tell-and-practice condition or an invent-with-contrasting-cases condition. In the tell-and-practice condition, students were told the relevant concepts and formulas on density and then practiced with contrasting cases. In the invent-with-contrasting-cases condition, students had to invent formulas with the same contrasting cases first, and then formulas were provided only after they completed all the inventing tasks. Both groups of students showed a similar level of proficiency at applying a density formula on a word problem; however, the invention students showed better performance on the transfer tests that also required an understanding of ratio concepts but had semantically unrelated topics. Schwartz et al. (2011) argued that the tell-and-practice students did not have a chance to find the deep structure because they simply focused on what they had been told and practiced applying the learned formulas. Similar to the findings of Schwartz & Martin (2004), the inventing activity appeared to serve as preparation for future learning, and thus when the expert solutions were provided later, these students could appreciate the expert solutions better than those who were not prepared.

Kapur (2011, Kapur & Bielaczye 2011) also tested the effects of combining invention activities with follow-up instruction in multiple classroom studies and showed that it was indeed more effective than just providing direct instruction without invention activities. Kapur (2008) explains this with what he calls productive failure. Even though most students fail to generate valid methods on their own during the invention phase, this failure experience actually helps students become prepared to learn better in the following learning phase by activating students' prior knowledge and having students attend to critical features of the concept.

Based on the idea that invention activity can bolster learning when combined with a follow-up instruction, Roll and his colleagues (2010) integrated the strengths of exploration/invention with strengths of direct instruction in a computer-based tutor called the Invention Lab. In the Invention Lab, students are given invention tasks where they have to invent novel methods for computing certain properties of data. Roll et al. (2011) found that students who both designed and evaluated their own methods performed better than those who only evaluated methods without design activity on conceptual knowledge and debugging tests.

SUMMARY

We have reviewed several decades of debates and empirical evidence on discovery learning approaches and direct instructional approaches. Both positive and negative effects have been reported. Positive effects of discovery learning have been reported in alternative classroom projects where students were encouraged to invent their own procedure to solve a problem and discuss their solutions with their classmates. High levels of practice also facilitate successful discovery learning. Minimal guidance is known to have several cognitive benefits such as better memory and transfer as a result of the generation effect and to develop better attitudes toward a learning domain. However, it can be disadvantageous when, as a result of unnecessary excessive floundering, students fail to discover principles they are expected to learn.

On the other hand, a number of empirical studies suggest positive effects of providing direct instruction. Strong empirical evidence is especially found in example-based learning, although one can question whether this should be characterized as direct instruction. When students are given step-by-step solutions, they are known to learn better than when they simply practice problem solving. According to the cognitive load theory, worked examples help students focus on relevant problem solution steps by reducing irrelevant search activity such as means-ends analysis that is mostly found when solving unfamiliar problems. In contrast to the strong empirical support for worked examples, it is not still clear whether adding explanations to worked examples helps learning or not. It appears that extra explanation is only helpful when it is appropriately embedded into a worked example in a way that allows learners to integrate multiple sources of information without burdening their working memory. When multiple sources of information fail to be integrated, it causes a split-attention effect and thus hinders learning.

We have also reviewed other instructional factors that might influence the effectiveness of instructional methods. Learner characteristics, such as prior knowledge, interact with levels of instruction. The provision of guidance is sometimes not beneficial for advanced learners (i.e., expertise reversal effect), whereas it helps inexperienced learners. There is strong support to suggest that self-explanation helps learning through the generation process. Comparing multiple solutions also increases the effectiveness of worked examples, especially when learners have sufficient prior knowledge to make an analogy from one to the other.

Several attempts have been made to combine strengths of both discovery learning and direct instruction approaches. Worked examples and problem-solving practice were successfully combined using a fading procedure (e.g., Atkinson et al. 2003, Renkl et al. 2002, Salden et al. 2010). When transitioning from worked examples to problem-solving practice depending on the stage of learning, it led to better learning outcome than administering just one instructional method. Following suggestions from the preparation for future learning method, having students experience an exploration phase followed by an instruction phase also led to better learning. Productive failure can take advantage of the strengths of both the discovery learning and direct instruction approaches.

CONCLUSIONS

Although there are islands of clarity in this field, it is apparent that there is not a comprehensive understanding that would predict the outcome of different amounts of guidance across different learning situations. We think the fundamental reason for this is that despite all the pronouncements, there is not a detailed understanding of the mechanisms by which students turn their learning experiences into knowledge.

We have mainly focused on domains where the target competence is the ability to solve problems. In such domains it rarely (if ever) happens that students can simply take the words they hear from a teacher or find on a page and convert these into the sort of knowledge that they can transfer. In some way students must construct the knowledge by understanding how it applies to their problem solving. It is also a rare case that the best way for students to achieve such an understanding is by being left to figure it out entirely for themselves. Acquiring knowledge the first time this way took centuries. The key question is how students can be guided to construct this knowledge efficiently in a form that will transfer across the desired range of situations.

Looking back on this review, we are struck by two things. First, there is relatively little evidence (but not none) that verbal instruction helps. Second, there seems to be a great abundance of evidence that providing an example of a problem solution does help. We are tempted to believe that pure discovery learning succeeds only because successful discovery can provide the student with examples to learn from, which they have come to understand through the discovery process. We are equally tempted to believe that pure verbal instruction is effective only to the extent that it helps students understand real or imagined examples. That is, we suspect that learning in problem-solving domains is fundamentally example based and that both instruction and discovery have their effects in helping students understand the examples.

While the acquisition of problem-solving competence may be example based, we have also reviewed ample evidence that not all examples are equally effective and that what accompanies these examples can be critical. The most important role of verbal instruction may be to draw attention to the critical aspects of the examples. It is also important to do this in a way that is efficient and does not burden the student with unnecessary processing. It is often possible to achieve the same effect by nonverbal highlighting mechanisms. The sequencing and juxtaposition of examples can serve a role similar to highlighting critical features. If students can solve the problem on

Erratum

Table 1 Advantages and disadvantages of providing instruction

Advantages	Disadvantages
• Provides correct solutions and explanations	• Solution methods may be rotely learned and poorly remembered
• Guides students to material to be learned	• Discourages learning that goes beyond the instruction
• Identifies critical features in the examples	• Prevents students from testing the adequacy of their understanding
• Makes time efficient by reducing floundering and irrelevant search	• Processing verbal instruction can pose a comprehension burden
• Reduces working memory demands created by managing problem solving	• Splits attention when multiple sources of information are not integrated

their own without guidance, this can be an effective way to identify what is critical about the example solution that is generated.

With this perspective in mind, we have put together **Table 1**, which summarizes some of the possible advantages and disadvantages from providing instructional guidance to the learner.[2] The biggest advantage of instruction is that it provides learners with correct information that may never be found by learners on their own. On the other hand, this information may be only rotely memorized and poorly remembered. Instruction will focus the student on critical material and pull their focusing away from the irrelevant, but it will also discourage types of learning that may be more useful. When studying an example, the instruction can highlight the critical features but can also prevent students from testing whether they really understand what is critical in the example. Typically, instruction will prevent floundering, but processing the instruction itself can be a time sink. Finally, problem solving on one's own can take resources away from learning, but so can trying to integrate the multiple sources of information that are frequently part of instruction.

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[2]A similar table on the benefits and cost of assistance giving and withholding is found in Koedinger & Aleven (2007).

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